



2nd International Conference on Sustainable Civil Engineering Structures and Construction
Materials 2014 (SCESCM 2014)

Sizing, shape, and topology optimizations of roof trusses using hybrid genetic algorithms

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Abstract

Structural optimizations have received great attention from structural engineers. Several optimization methods have been proposed including evolutionary strategies and genetic algorithms. This paper considers hybrid genetic algorithms for roof truss optimizations. Practically, roof truss optimizations are unique. In this case, the pitch angles are usually governed by roof covering types. In the optimization process, the pitch angle is set to constant, while the coordinates of the joints are determined by genetic algorithms. The optimization process utilizes hybrid genetic algorithms, i.e., a combination of binary and real coded genetic algorithms. Genetic algorithms are optimization methods that have been used successfully for various problems. For the sizing, shape and topology optimizations considered in this paper, the area of cross section and the number of members connected to every node are optimized using binary coded genetic algorithms, while the coordinates of the nodes are determined using real coded genetic algorithms. The optimization process for binary and real coded algorithms is done subsequently. The use of real coding for joint coordinates of structures gives the program the flexibility to obtain the final position of the joints. The arithmetic crossover is used to tackle this matter. In every generation, a portion of new individuals is inserted randomly replacing the old individuals. This can be considered to increase the variability of the population. In addition, the fittest individual is always transferred into the next generation. The penalty to the individuals that are violating the constraint is set to a minimum fitness in this paper. It can be shown that the proposed procedure is able to obtain the optimum design of roof truss structures.

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Peer-review under responsibility of organizing committee of the 2nd International Conference on Sustainable Civil Engineering Structures and Construction Materials 2014

Keywords: optimizations; sizing optimizations; shape optimizations; topology optimizations; hybrid genetic algorithms; truss structures

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1. Introduction

Sizing, shape, and topology optimizations are important in structural design. The objective of sizing optimization is to obtain the cross section that produces the least weight given the shape and the topology of the structures. In shape optimizations the coordinates of the node are to be sought in order to obtain the optimum structures. In addition, in topology optimizations the number of nodes and how the members are connected to nodes are important. Therefore, it is clear that sizing, shape and topology optimizations are complex problems.

Recently, Genetic Algorithms (GAs) are widely used for solving many optimization problems [1-7], including structural optimizations [8-13]. GAs search for the solution by initializing a population of random candidates. These candidates experience evolutionary processes based on survival-of-the-fittest mechanisms. After crossovers and mutations, a new population is created based on the previous individuals through a certain selection procedure. The individuals that have superior fitness naturally will be passed to the next generations. Elitist strategies [14] are used to assure that the best individuals will survive into the next generation. In addition, after crossovers and mutations, new individuals are inserted to replace a number of old individuals in the population. This is done to increase the variability of the populations [5].

2. Hybrid genetic algorithms

2.1. GAs for structural optimizations

In the early development of GAs, binary-coded GAs were widely used by researchers. Recently real-coded GAs have been developed for solving optimization problems, which are considered to be more efficient compared to binary-coded GAs. For sizing, shape, and topology optimization, hybrid GAs are used to tackle this problem. In this case both binary and real coding are utilized together to solve the problem.

In binary-coded GAs, the chromosomes are represented by number 0 and 1, which then are converted to integers or real numbers. The length of the strings defines the value of the integer or real number. On the other hand, the chromosomes in real-coded GAs are directly represented by real numbers. Depending on the objective of the problems, both GAs have their own advantages for the problems considered.

The GA program developed in [4] is used in this paper. The binary and real coding are then combined following [6].

2.2. Sizing optimizations

For sizing optimizations, the shape and the number of members are usually defined by the designers based on their experience and structural design limitations. The chromosome that is represented by a string containing 0 and 1 is converted to an integer number by using

$$t_i = \sum_{j=0}^r h_j \times 2^j \quad (1)$$

where h_j ($j=0..r$) = a binary string containing 0 or 1, r = the length of the string. The resulting integers represent discrete sections initially supplied by designers.

2.3. Topology optimizations

In topology optimization, the way and how many members are connected to each node are sought. The possible existing members are represented by using

$$j_b = (node-1) \times 0.5 \times node \quad (2)$$

where j_b = possible existing members, and $node$ = number of nodes in the structure. The binary strings containing 0 and 1 are used to indicate whether the member is connected or not to the nodes. As discussed by [12], consider a structure with four nodes as shown in Fig. 1 with all possible members connected to the nodes according to Eq. (2). When the resulting binary string is [0 1 1 0 0 1] then there are second, third and sixth members available. The layout of the structure for this case is depicted in Fig. 2.

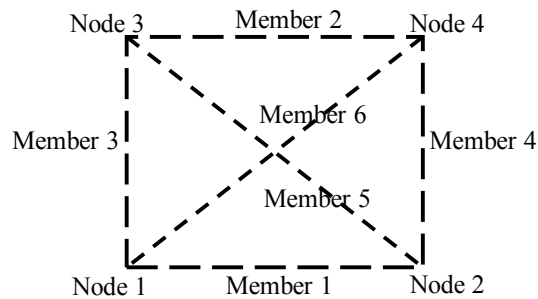


Fig. 1. Four-node structure.

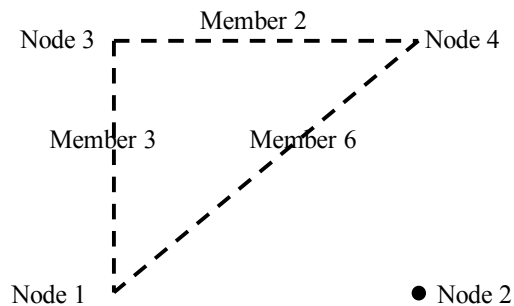


Fig. 2. Layout of the structure represented [0 1 1 0 0 1] string.

2.4. Shape optimizations

In shape optimizations, the location of the nodes is optimized. In this case real-coded GA is suitable to obtain the coordinates of the nodes. In real coding GAs, real numbers are used to represent the design variables. For the crossover, arithmetic crossover is used so that it has the ability to explore a larger domain of solution [5].

2.5. Hybrid genetic algorithms

This paper considers hybrid-coded genetic algorithms (hybrid-coded GAs) similar to the ones in [6] for optimization of the location and properties of the tuned mass dampers. Here hybrid-coded GAs are utilized to optimize the roof truss structures. The binary and real-coded GAs are employed jointly in one code. The binary-coded GAs are used to optimize the size of members and the topology of the truss, while real-coded GAs are used to obtain proper locations of structural nodes.

2.6. Fitness, constraints and penalty functions

The objective function used in structural optimization is the total weight of the structures as

$$W = \sum_{i=1}^k \rho A_i l_i \tag{3}$$

where W = weight of structure, ρ = density of members, A_i = cross section of member-i, and l_i = length of member-i.

Since in GAs the objective is to maximize the fitness of the chromosomes, the objective function is converted to a fitness function as

$$F = C \times \frac{1}{W} \tag{4}$$

where F = fitness function, and C = the scaling factor.

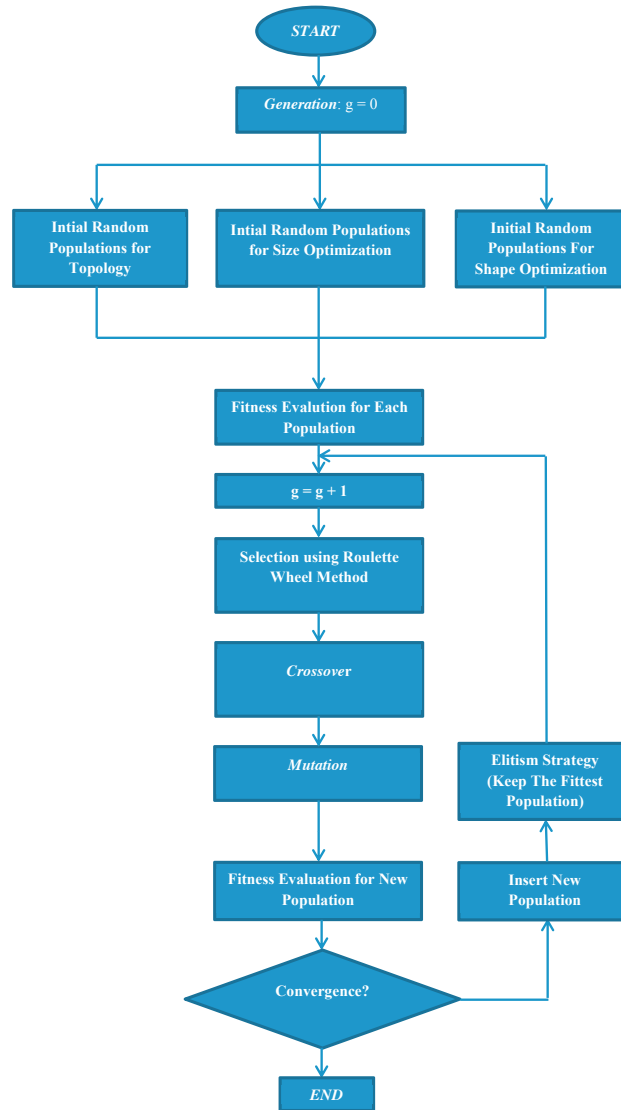


Fig. 3. Flowchart of hybrid GAs for topology, size and shape optimizations

In this roof truss optimization, there are three constraints taken in this paper: stress, displacement and slenderness. In the optimization process, the design variables that are violating the constraints are penalized by

assigning a very small value of their fitness that can be accepted by computers. A Matlab program developed in [4] and [6] is used to optimize the structure. The flowchart of the program is depicted in Fig. 3.

The selection of individuals is done by using a roulette wheel selection procedure. For real-coded GAs, arithmetic crossover is used, while for binary coding ones one-point crossover is employed. In this GA after selection, mutation and crossover a portion of newly fresh individuals is inserted into population [4,5] so as to increase the variability of the population.

3. Numerical examples

3.1. Validation

Before applying hybrid GAs to optimize the size, shape and topology a benchmark problem [8] is taken as a test structure. The structure is shown in Fig. 4, where $P = 445.374$ kN (= 100 kips). The material properties are: $E = 68.95$ GPa, $\rho = 2768$ kg/m³, the compression and tension stresses are limited to 172.37 MPa, and displacement constraint is 50.8 mm for both horizontal and vertical displacements. The buckling is ignored in stress calculation.

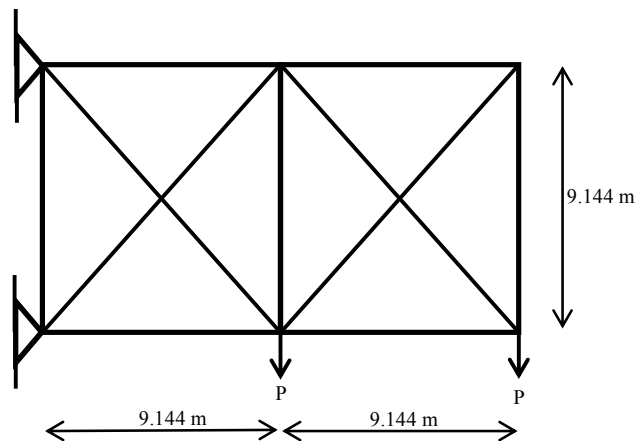


Fig. 4. A benchmark problem.

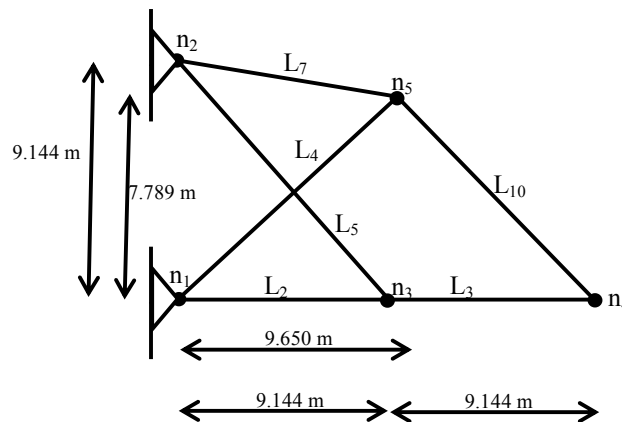


Fig. 5. The result of optimization.

Hybrid-coded GAs are used to optimize the structure, where the population size = 20, the crossover rate = 0.8, and mutation rate = 0.1. Structural analysis is done by using a program developed in [15,16]. The result of the optimization is shown in Table 1. The resulting layout of the structure according to hybrid-coded GAs can be seen in

Fig. 5, while the results of the cross section are shown in Table 2. The history of the individual with the highest fitness level is shown in Fig. 6.

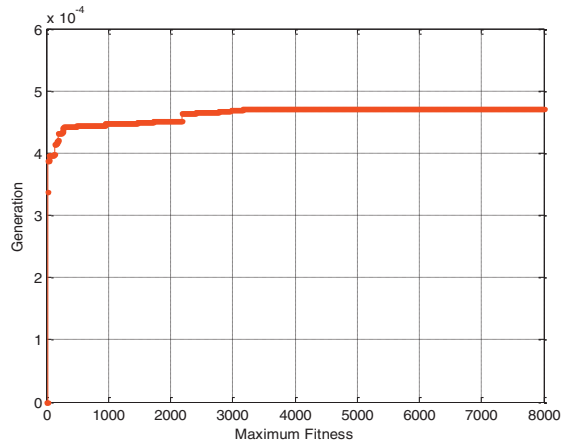


Fig. 6. Best fitness history.

It can be seen from Table 1 that the hybrid-coded GAs outperform other available results.

Table 1. Results of the test structure.

Proposal	Weight (kg)	Method
Deb and Gulati [17]	2222.22	Size and topology GAs
Hajela and Lee [18]	2241.97	Size and topology GAs
Li. Huang and Liu [19]	2295.59	Size optimization, PSO
Kriparan, Gupta and Baugh Jr. [20]	2301.09	Size optimization, hybrid search method
Galante [21]	2322.08	Size and shape optimization GAs
This paper	2122.622	Size, shape and topology GAs

3.2. Size, shape and topology optimizations

In this paper, roof truss optimization using hybrid coding GAs are considered. Practically, roof truss optimizations are unique. In this case, the pitch angles are usually decided by designers and governed by the roof covering types. In the optimization process, the pitch angle is set to constant, while the coordinates of the joints are determined by real-coded GAs. The optimization of the size of the member's cross section and whether the members are connected to two nodes or not are optimized using binary coding GAs. Here the binary and real-coded GAs are employed simultaneously to form hybrid-coded GAs.

Table 2. Results of optimization of the test structure.

Member	Start coordinate	End coordinate	A (mm ²)	L (m)	M (kg)	Stress (N/mm ²)	Displacement (mm)
2	(0;0)	(9144;0)	4620	9144	116.9349	-96.4	48.9446
3	(0;0)	(18288;0)	10900	18288	551.7709	45.3	50.7995
4	(0;0)	(9650;7789)	12200	12401	418.7768	41.8	18.8784
5	(0;9144)	(9144;0)	4620	12931.6	165.371	-136.3	43.6491
7	(0;9144)	(9650;7789)	14700	9744.7	396.5079	61.2	18.3884
10	(9650;7789)	(18288;0)	14700	11631	473.2607	45.2	45.7754

3.3. Numerical example 1

The first application of roof truss optimization is considered in this example as shown in Fig. 7. The span length of the truss is 10 m, and the pitch angle is taken so that the height of the truss is 3 m. The locations of nodes 1, 3 and 4 are fixed, while nodes 5, 6, 7, and 8 are sought to obtain the optimum structure. Point loads at nodes 2, 4, 5, 6, 7, and 8 are taken to be equal to 200 kg. The cross section of symmetrical angle supplied for the optimizations are [1410 1670 1230 1510 1790 2060 1550 1870 2180 1920 2270 2620 2120 2510 2900 2540] mm².

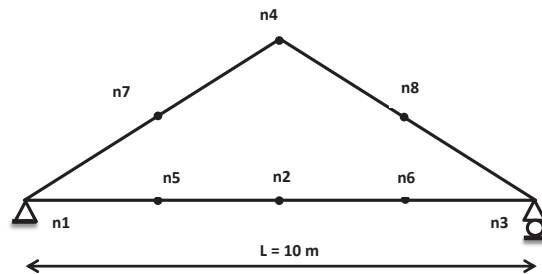


Fig. 7. First roof truss application.

Hybrid coded GAs similar to the previous ones are used to optimize the shape, topology and size of the structural members. In this case, the population size = 20, maximum generation = 2000, probability of crossover = 0.8, and mutation rate = 0.1. The optimization of size, shape and topology can be seen in Fig. 8 and Table 3. The second run with a population size of 30 gave the same result as in the first one. The evolving best fitness for a population of 30 can be seen in Fig. 9. The optimum structure has a total weight = 325.9103 kg, maximum stress = 0.0198 kN/mm², and maximum displacement = 0.8531 mm.

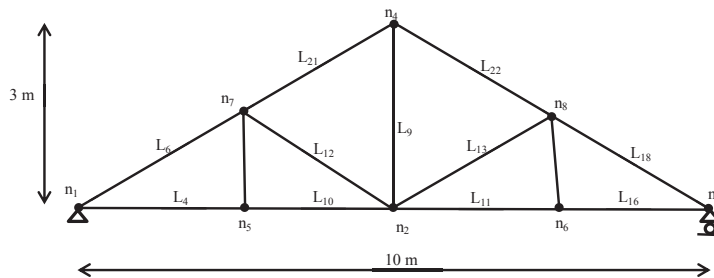


Fig. 8. Result of optimum structure.

Table 3. Results of optimization of example 1.

Member	Start coordinate	End coordinate	A (mm ²)	L (m)	M (kg)	Stress (N/mm ²)	Displacement (mm)
1	(0;0)	(3000;0)	1230	3000.000	28.2285	0.0073	0.4166
4	(0;0)	(1291.1;865.037)	1230	1554.100	14.6233	0.0111	0.3826
8	(3000;0)	(6000;0)	1230	3000.000	28.2285	0.0073	0.4166
10	(3000;0)	(1291.1;865.037)	1230	1915.400	18.0230	0.0022	0.3224
11	(3000;0)	(2083;1395.61)	1230	1669.900	15.7129	0.0016	0.4080
12	(3000;0)	(3852.3;1438.959)	1230	1672.400	15.7364	0.0019	0.3061
13	(3000;0)	(4804.9;800.717)	1230	1974.500	15.7364	0.0023	0.4250
18	(6000;0)	(4804.9;800.717)	1230	1438.500	18.5791	0.0108	0.2549
20	(3000;2000)	(2083;1395.61)	1230	1098.300	13.5356	0.0017	0.3942
21	(3000;2000)	(3852.3;1438.959)	1230	1020.400	10.3345	0.0016	0.2902
23	(1291.1;865.037)	(2083;1395.61)	1230	953.2121	9.60150	0.0077	0.3934
26	(2083;1395.61)	(3852.3;1438.959)	1230	1769.800	8.96920	0.0074	0.3867
28	(3852.3;1438.959)	(4804.9;800.717)	1230	1146.60	8.96920	0.0080	0.2849

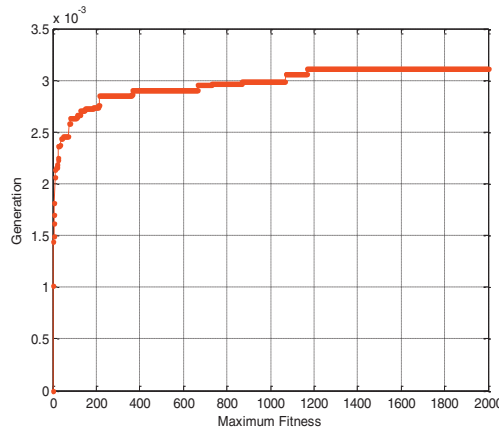


Fig. 9. Evolving best fitness for example 1.

In the previous example the point loading in the nodes is taken as constant. Here we also try to optimize the structure when the point loading is changed depending on the location of the nodes. It is assumed that the distance between columns supporting the truss in the direction perpendicular to the truss is 6 m, covering roof and ceiling loading = 50 kg/m². However, due to live load it is assumed that the point loading at each node is 200 kg. The optimum result is the same as in the previous case.

3.4. Numerical example 2

A second numerical example is taken as shown in Fig. 10, where the height of the truss = 2 m. Again, four nodes have fixed locations, i.e., nodes 1, 2, 3, and 4.

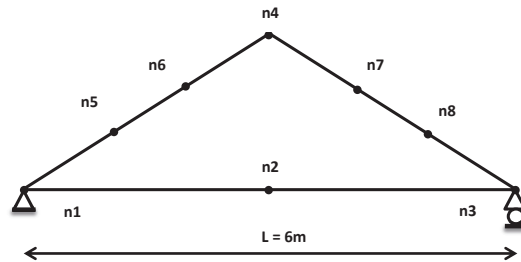


Fig. 10. Example 2.

The same GAs are used to optimize the shape, topology and the cross sectional area. The resulting optimization is shown in Fig. 11 and Table 4, while the best fitness history is shown in Fig. 12. The resulting cross section is 1230 mm² for all members, which is the smallest from the list, where the total weight of the structure is 206.2781 kg.

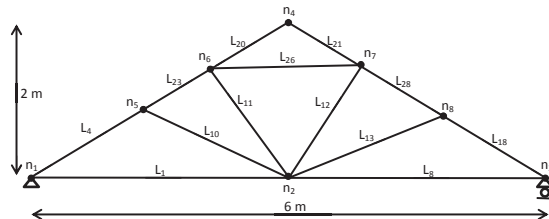


Fig. 11. Optimum structure example 2.

Table 4. Results of optimization of example 2.

Member	Start coordinate	End coordinate	A (mm ²)	L (m)	M (kg)	Stress (N/mm ²)	Displacement (mm)
1	(0;0)	(3000;0)	1230	3000.000	28.2285	0.0073	0.4166
4	(0;0)	(1291.1;865.037)	1230	1554.100	14.6233	0.0111	0.3826
8	(3000;0)	(6000;0)	1230	3000.000	28.2285	0.0073	0.4166
10	(3000;0)	(1291.1;865.037)	1230	1915.400	18.0230	0.0022	0.3224
11	(3000;0)	(2083.1395.61)	1230	1669.900	15.7129	0.0016	0.4080
12	(3000;0)	(3852.3;1438.959)	1230	1672.400	15.7364	0.0019	0.3061
13	(3000;0)	(4804.9;800.717)	1230	1974.500	15.7364	0.0023	0.4250
18	(6000;0)	(4804.9;800.717)	1230	1438.500	18.5791	0.0108	0.2549
20	(3000;2000)	(2083;1395.61)	1230	1098.300	13.5356	0.0017	0.3942
21	(3000;2000)	(3852.3;1438.959)	1230	1020.400	10.3345	0.0016	0.2902
23	(1291.1;865.037)	(2083;1395.61)	1230	953.2121	9.60150	0.0077	0.3934
26	(2083;1395.61)	(3852.3;1438.959)	1230	1769.800	8.96920	0.0074	0.3867
28	(3852.3;1438.959)	(4804.9;800.717)	1230	1146.600	8.96920	0.0080	0.2849

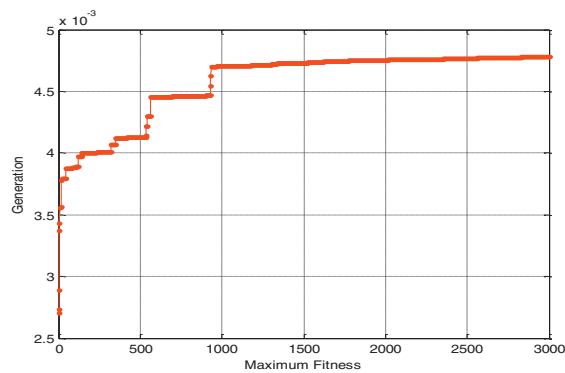


Fig. 12. Evolving best fitness example 2.

3.5. Numerical example 3

The third example is a structure as shown in Fig. 13. The span length = 25 m, height of truss = 3 m. Again, the location of three nodes, i.e., nodes 1, 5, and 10, are set to a fixed location. Nodes 2, 3, 4, 6, 7, 8, and 9 are optimized by the program. The section properties are taken as $A = [3500 \ 3930 \ 4000 \ 4030 \ 4500 \ 4570 \ 4610 \ 5100 \ 5180 \ 5540 \ 5750 \ 6180 \ 6190 \ 6840 \ 6910 \ 7640]$ mm².

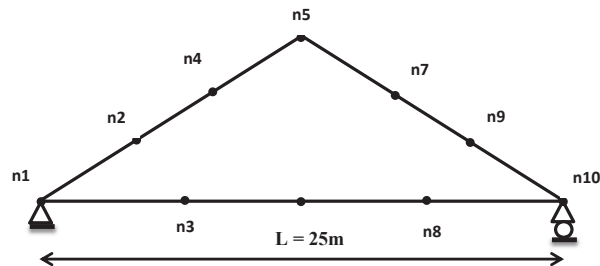


Fig. 13. Example 3.

The same hybrid-coded GAs are used to optimize the structure. The resulting structure is shown in Fig. 14 and Table 5. Here the smallest section properties are obtained for all members. The optimum weight of the structure is 1956.0637 kg, maximum stress is 10 N/mm², and maximum displacement is 0.6685 mm.

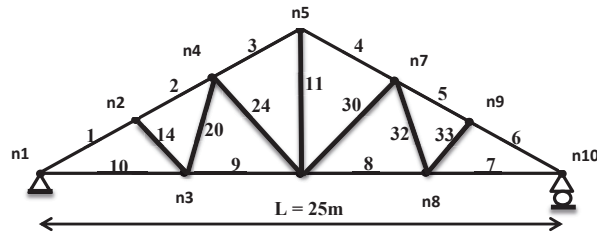


Fig. 14. Resulting shape and topology optimization.

Table 5. Results of optimization example 3.

Member	Start coordinate	End coordinate	A (mm ²)	L (m)	M (kg)	Stress (N/mm ²)	Displacement (mm)
1	(0;0)	(4429;1062.96)	3500	4554.8	121.9548	1.6000	0.55
2	(4429;1062,96)	(8063;1935.12)	3500	3737.2	100.0635	0.7043	0.6685
3	(8063;1935.12)	(12500;3000)	3500	4563.0	122.1743	0.5275	0.6685
4	(8063;1935.12)	(17980;1924.8)	3500	5584.5	149.5250	0.2833	0.5290
5	(17980;1924.8)	(20192.2273;1153.9135)	3500	2342.7	62.72580	0.4074	0.3880
6	(20192,2273;1153.9135)	(25000;0)	3500	4944.3	132.3836	0.8897	0.3305
7	(18750,0)	(25000,0)	3500	6250.0	167.3438	2.3000	0.4472
8	(12500;3000)	(18750,0)	3500	6250.0	167.3438	4.2000	0.5731
9	(6250;0)	(12500;0)	3500	6250.0	167.3438	7.8000	0.5815
10	(0;0)	(6250;0)	3500	6250.0	167.3438	10.000	0.5815
11	(12500;3000)	(12500;3000)	3500	3000.0	80.32500	0.0158	0.5734
14	(4429;1062.96)	(6250;0)	3500	2108.5	56.45510	0.1928	0.4438
20	(6250;0)	(8063;1935.12)	3500	2651.7	70.99930	0.1662	0.5742
24	(8063;1935.12)	(12500;0)	3500	4840.6	129.6071	0.3428	0.5159
30	(12500;0)	(17980;1924.8)	3500	5808.2	155.5146	1.1000	0.6089
32	(17980;1924.8)	(18750,0)	3500	2073.1	55.50730	0.0791	0.5105
33	(18750,0)	(20192.2273;1153.9135)	3500	1847.0	49.45340	0.1309	0.5082

4. Conclusions

Sizing, shape, and topology optimization using hybrid-coded GAs have been considered in this paper. The location of nodes is optimized by using real-coded GAs while the size and whether the members are connected to the nodes are optimized using binary-coded GAs. The hybrid-coded GAs has been validated by comparing hybrid GAs used in this paper with other methods for problem benchmarking of a truss structure. The hybrid-coded GAs are then used to optimize the size, shape and topology of the roof structures. Three examples of size, shape and topology optimization of roof truss structures have been shown to demonstrate the applicability of the hybrid-coded GAs.

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