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MOMENT COEFFICIENTS FOR STATICALLY INDETERMINATE PRESTRESSED CONCRETE STRUCTURES

Y. Arfiadi[a] and M.N.S. Hadi[b*]
[a]Department of Civil Engineering, Atma Jaya Yogyakarta University, Indonesia
[b]Faculty of Engineering, University of Wollongong, Australia

ABSTRACT

In statically indeterminate prestressed concrete structures, prestressing force produces secondary moment in addition to primary moment due to eccentricity. This condition is different from statically determinate structures where there is no secondary moment effect and the moment due to prestressing is due to primary moment only, i.e., prestressing force times eccentricity. With the presence of secondary moment, prestressing force design becomes more complex, because the secondary moment is a function of prestressing force and the geometry of the structures. In addition, considering that in general the cable profile is parabolic or another type of curves, which also occurs at continuous supports, the load balancing method may not be used. To cope with this problem moment due to prestressing force is assumed to be the prestressing force times a β coefficient. In statically determinate structures the β coefficient equals the cable eccentricity to the center of gravity of the section. Therefore, the β coefficient can be considered as a statically indeterminate eccentricity. By assigning that the moment due to prestressing force as a function of prestressing force and by considering the allowable stress requirements at top and bottom fibers, equations can be derived to compute the prestressing force in statically indeterminate structures. From the derived equations, the upper and lower bounds of prestressing force can be determined if the section satisfy the requirements. If the optimum prestressing force is needed, the difference of lower and upper bounds should be minimum. Nevertheless, the difference of lower and upper bounds can be considered as a safety level. At the end of the paper examples are presented to show the application of the proposed method.

Keywords: Prestressed concrete; prestressing force; statically indeterminate structures; secondary moment; moment coefficient

1. INTRODUCTION

Prestressed concrete structures have become an alternative way of design in order to obtain sophisticated and economical structures particularly for long span concrete structures. With

* E-mail address of the corresponding author: mhadi@uow.edu.au (Muhammad Hadi)

the simple principles to give compression to the concrete so that during the service load the tensile stress will be eliminated while limiting the compressive stress within the prescribed value, prestressed concrete has become an attractive approach in concrete structures design. However, some difficulties may arise in designing prestressed concrete members for statically indeterminate structures. This is due to the presence of secondary moment when the prestressing force is applied [1-5]. The interaction between the secondary moment and the magnitude of prestressing force produces more challenging tasks, because the magnitude of the secondary moment might be significantly large enough and can not be neglected.

When Lin's load balancing method [1] is used, some conditions should be satisfied. First, the cable profile is assumed curved or parabolic in each span of the member with no smooth transition. Therefore, the drastic changes in the cable profile in continuous supports are neglected in the computation so that the prestressing force can balance a part of the external loading in every span of member. Besides that we can not have cable eccentricities at the end supports as those eccentricities produce additional moments. This condition results in that when the prestressing force is obtained by using load balancing method, we have to check the stress to account for those two conditions.

Another method to handle the secondary moment is by designing the cable profile so that the cable is concordant, i.e., the C-line (the profile of the line of thrust) coincides with the T-line (the profile of tendon), in the case without external loading [3, 4]. By using this procedure the secondary moment would be zero. However, obtaining such profiles is not an easy task and needs some procedures to transform the tendon profile.

In this paper a simple procedure to obtain the magnitude of prestressing force in statically indeterminate concrete elements is proposed. By assuming that the total moment due to prestress as a linear function of the magnitude of prestressing force, and employing the relationships between stress limitation, the magnitude of prestressing force can be obtained. The inequalities then can be solved defining the lower and upper bounds of the prestressing force so that when such prestressing force is applied to the members, the stress will be in the prescribed limit, with the secondary moment has been taken into account. With this procedure the determination of prestressing force will be simple. In addition, this method can be considered as a general procedure that can be used either for statically determinate or indeterminate structures. In statically determinate prestressed concrete structures, the value of secondary moment would be zero. It is to be noted that the economical design will be accomplished when the difference between lower and upper magnitudes of prestressing force is small. The difference between the lower and upper bound magnitudes of prestressing force can be considered as "a degree of safety".

2. SECONDARY MOMENT

In statically determinate structures, the moment due to eccentricity is the same as the moment due to the effect of equivalent load due to prestressing force. As can be seen from Figure 1, the moment due to the eccentricity of the tendon to the center of gravity of the section at any point is

$$M_1 = Fe \tag{1}$$

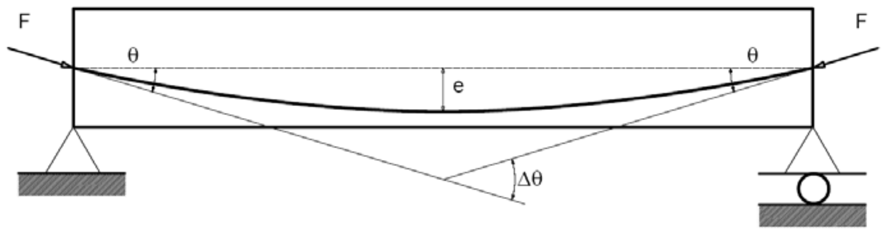

Figure 1. Statically determinate structures

Due to prestressing, the transverse equivalent load is generated along the beam's span. The force at the end of the beam can be seen in Figure 2 (a) which can be simplified as follows:

(a) Vertical component of force = F $\sin\theta$, if θ is small the vertical component becomes $= F\theta$,

(b) Horizontal component of force = F $\cos\theta$, if θ is small the horizontal component becomes = F.

The moment due to prestressing force is depicted in Figure 2(b).

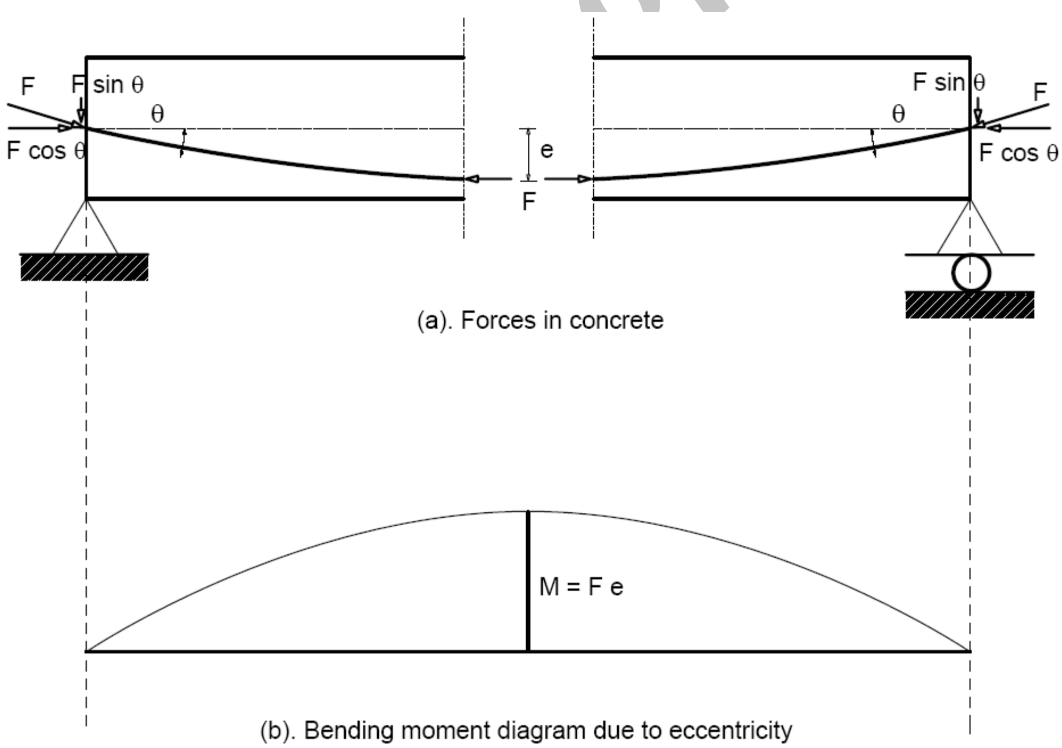

Figure 2. Force due to prestressing and bending moment due to eccentricity of prestressing force

If the tendon's trajectory is curved it produces an equivalent uniform load = q to the beam as shown in Figure 3(a). The total load should be the same as the vertical component of prestressing at ends of beam in Figure 2 (a). The equilibrium of vertical forces results in

$$q \times L = 2 \times F\theta \tag{2a}$$

so that

$$q = \frac{2F\theta}{L} \tag{2b}$$

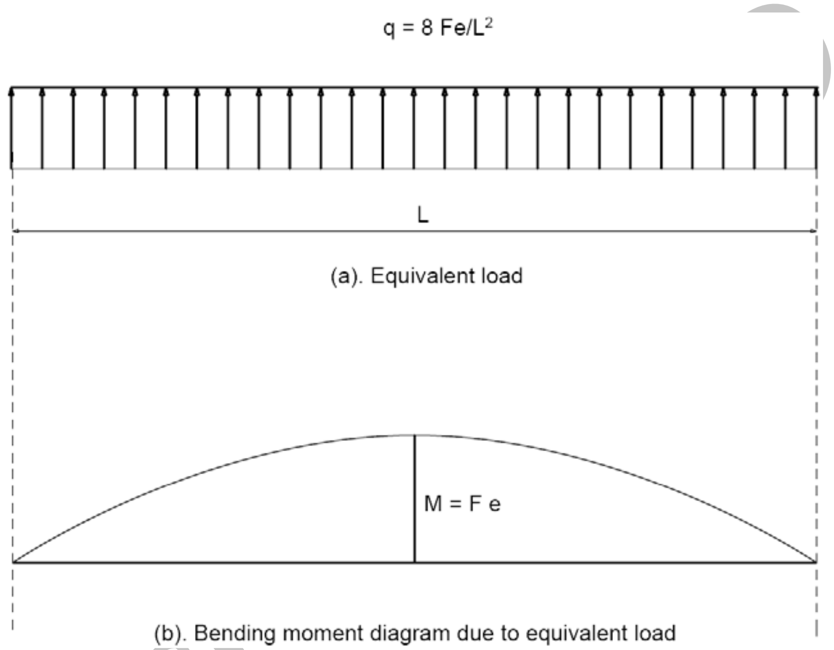

Figure 3. Equivalent load and moment due to equivalent load in statically determinate beam

When the trajectory is parabolic, the ordinate of the tendon can be expressed as

$$y = \frac{4ex(L-x)}{L^2} \tag{3}$$

so that

$$\theta = \left(\frac{dy}{dx}\right)_{x=0} = \frac{4e}{L} \tag{4}$$

and the vertical load becomes uniformly distributed as

$$q = \frac{8Fe}{L^2} \tag{5}$$

Moment due to equivalent uniform load in the middle span is

$$M_2 = \frac{qL^2}{8} \tag{6a}$$

and by substituting Equation (5) into (6a) results in

$$M_2 = Fe \tag{6b}$$

It can be seen that moment due equivalent load (M_2) in equation (6b) is the same as moment due to prestressing force and eccentricity (M_1) in Equation (1).

Consider now a hypothetical indeterminate structure as shown in Figure 4. Moment due to prestressing force and eccentricity M_1 is the same as in Equation (1), i.e., prestressing force times eccentricity. While moment due to eccentricity at beam ends is equal to zero, assuming that there is no eccentricity between tendon profile and center of gravity of the section. The bending moment diagram due to prestressing and eccentricity is the same as in Figure 2 (b).

Since the tendon's trajectory in Figure 4 is the same as the one in Figure 1, it produces the same uniform loading q. Therefore, moment at beam ends due to equivalent uniform load in Figure 4 will be

$$M_2 = \frac{qL^2}{12} \tag{7a}$$

and by substituting Equation (5) results in

$$M_2 = \frac{2}{3}Fe \tag{7b}$$

Similarly the moment at midspan will be

$$M_2 = \frac{qL^2}{8} - \frac{qL^2}{12} = \frac{qL^2}{24} \tag{8a}$$

By substituting Equation (5) results in

$$M_2 = \frac{1}{3}Fe \tag{8b}$$

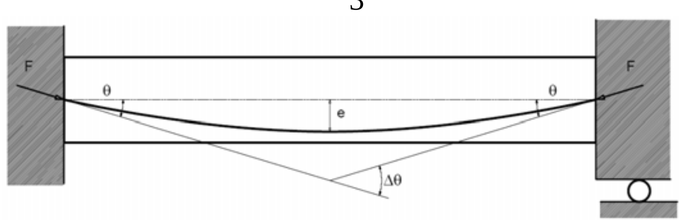

Figure 4. Statically indeterminate structure

The bending moment diagram due to uniform loading for the structure in Figure 4 is shown in Figure 5. If the bending moment diagram in Figure 5 is compared with the one in Figure 2 (b) or Figure 3, it is clear that, due to the uniform loading produced by prestressing force, both bending moment diagrams are different. If bending moment due to eccentricity is called primary moment, the difference between the moment due to equivalent load and the moment due to eccentricity is called the secondary moment, M_s, i.e.,

$$M_s = \Delta M = M_2 - M_1 \tag{9}$$

Therefore, for statically indeterminate structures, if only moment due to eccentricity is included, the secondary moment should be added in the stress calculation.

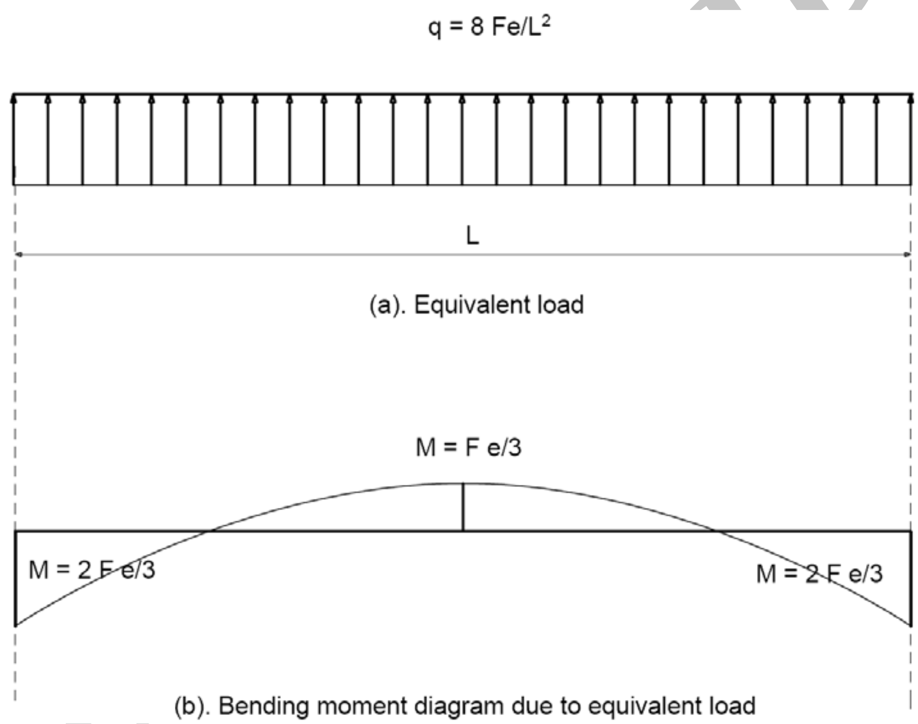

Figure 5. Moment due to uniform loading for the structure in Figure 4

Due to the presence of the secondary moment in statically indeterminate structures, the design becomes more complex. One of the methods to avoid this is by using load balancing method [1]. However, there are some limitations on using this method. One limitation is that the cable profile should have a smooth transition in the middle support for the case of continuous beams. In addition, in order to cancel the secondary moment, the eccentricity at the beam ends should be zero. Therefore, load balancing method is more appropriate to be used for statically determinate structures. Another method to avoid of calculating the secondary moment is by designing a concordant cable, where the C-line is designed to coincide with the T-line in the case of zero external loading. However, the design process becomes more complex.

3. MOMENT COEFFICIENT-β

In this paper the determination of prestressing force in statically indeterminate structures is obtained through the introduction of the moment coefficient due to prestressing force as β, i.e., the moment due to prestressing force at initial condition is

$$M_{Fi} = F_i \times \beta \tag{10a}$$

and assuming that the structure is linearly elastic we can obtain moment due to prestressing force at final condition as

$$M_F = F \times \beta \tag{10b}$$

where M_{Fi} = moment due to prestresssing force at initial (transfer), M_F = moment due to prestresssing force after loss of prestress, F_i = prestressing force at transfer and F= prestressing force after loss of prestress (the effective force). It is to be noted that for statically determinate structures the coefficient β turns to be eccentricity e. In view of that coefficient β can be considered as a ‘statically indeterminate eccentricity’.

Effective prestressing force at the final condition after loss of prestress has the relation

$$F = \alpha F_i \tag{11}$$

α = effective prestress coefficient after loss of prestress.

To obtain the moment due to prestressing some assumptions are taken as follows:

a) Cable eccentricity is small compared to the beam span;
b) Loss of prestress due to cable friction is neglected;
c) The number of cable is the same through the span length.

The determination of the equivalent load due to prestressing can be made with reference to Figure 6.

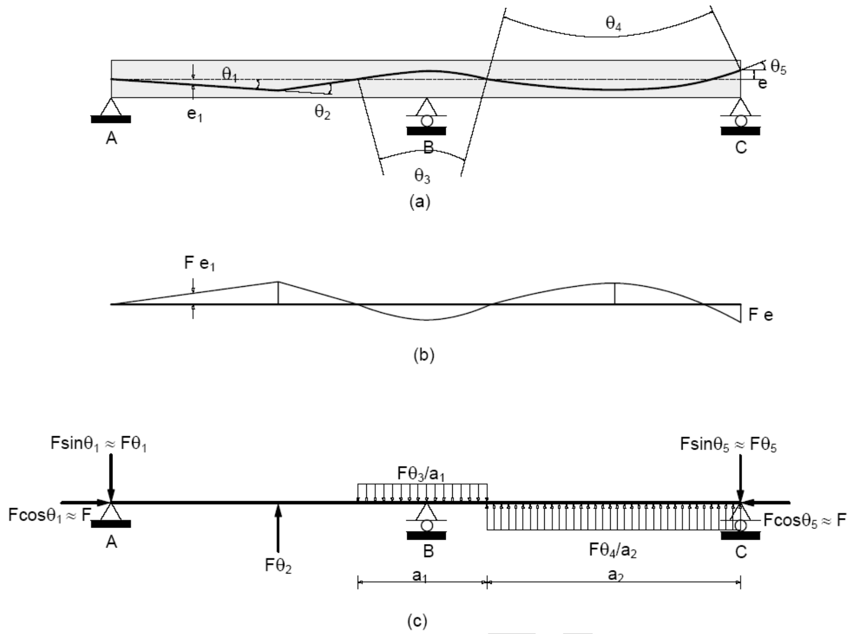

Figure 6. Computation of equivalent load and primary moment in beam: (a) beam with cable profile, (b) primary moment, (c) equivalent load due to prestressing

End A:
Horizontal load = $F\cos\theta_1 \approx F$,
Vertical load = $F\sin\theta_1 \approx F\theta_1$.
Span AB:
Vertical load point load = $F\theta_2$,
Vertical uniform load = $F\theta_3 / a_1$.

Span BC:
Vertical uniform load = $F\theta_3 / a_1$,
Vertical uniform load = $F\theta_4 / a_2$.

End C:
Horizontal load = $F\cos\theta_5 \approx F$,
Vertical load = $F\sin\theta_5 \approx F\theta_5$,
Moment = F e.

Having obtained the equivalent load on beam due to prestressing force, internal forces in beam can be obtained. The secondary moment can be computed by subtracting the moment due to equivalent load by the primary moment due to eccentricity. The coefficient β can be obtained from structural analysis by applying prestressing force as a unit force.

By assuming that:
(a) compression stress is assigned as negative (–) and tension stress is positive (+);
(b) positive moment when the bottom fiber is in tension;
(c) prestressing force F and F_i is assigned to be positive in the equation;

The stress in beam shall satisfy the provision provided in the building codes as follows.

At transfer (initial condition) at the top fiber if the stress is in tension:

$$-\frac{F_i}{A_c}-\frac{M_{Fi}\,y_t}{I_c}-\frac{M_{DL}\,y_t}{I_c}<\sigma_{ti} \tag{12a}$$

If the result is in compression:

$$-\frac{F_i}{A_c}-\frac{M_{Fi}\,y_t}{I_c}-\frac{M_{DL}\,y_t}{I_c}>\sigma_{ci} \tag{12b}$$

where A_C = area of section, I_c = second moment of area, y_t = neutral axis distance to top fiber, M_{DL} = moment due to dead load, σ_{ti} = allowable tension stress in concrete at transfer, σ_{ci} = allowable compression stress in concrete at transfer.

At bottom fiber if the stress is in tension:

$$-\frac{F_i}{A_c}+\frac{M_{Fi}\,y_b}{I_c}+\frac{M_{DL}\,y_b}{I_c}<\sigma_{ti} \tag{13a}$$

If the stress is in compression:

$$-\frac{F_i}{A_c}+\frac{M_{Fi}\,y_b}{I_c}+\frac{M_{DL}\,y_b}{I_c}>\sigma_{ci} \tag{13b}$$

where y_b = neutral axis distance to bottom fiber.

At the final stage (after loss of prestress) the following conditions shall be satisfied. At top fiber if the stress is in tension:

$$-\frac{F}{A_c}-\frac{M_F\, y_t}{I_c}-\frac{M_{TL}\, y_t}{I_c}<\sigma_t \tag{14a}$$

If the stress is in compression:

$$-\frac{F}{A_c}-\frac{M_F\, y_t}{I_c}-\frac{M_{TL}\, y_t}{I_c}>\sigma_c \tag{14b}$$

σ_t = allowable tension stress at the final condition, and σ_c = allowable compression stress at the final condition.

At bottom fiber if the stress is in tension:

$$-\frac{F}{A_c}+\frac{M_F\, y_b}{I_c}+\frac{M_{TL}\, y_b}{I_c}<\sigma_t \tag{15a}$$

If the stress is in compression:

$$-\frac{F}{A_c}+\frac{M_F\, y_b}{I_c}+\frac{M_{TL}\, y_b}{I_c}>\sigma_c \tag{15b}$$

In order to obtain the magnitude of prestressing force define first the following:

$$r=\sqrt{\frac{I_c}{A_c}} \tag{16}$$

$$Z_t=\frac{I_c}{y_t} \tag{17}$$

$$Z_b=\frac{I_c}{y_b} \tag{18}$$

Note that instead of using primary moment $F \times e$ or $F_i \times e$, moment due to equivalent load M_F or M_{Fi} is used in inequalities in Equations (12) – (15). The task of structural designers is to obtain the prestressing force F or F_i such that the stresses are less than the allowable stresses stipulated in building codes. This can be achieved by utilizing Equations (10a) and (10b). In order to obtain the coefficient β, the prestressing force can be assigned as a unit force in structural analysis.

4. PRESTRESSING FORCE BASED ON INITIAL CONDITIONS

(A). BASED ON ALLOWABLE STRESS AT THE TOP FIBER

If Equation (12a), i.e., when the stress at top fiber is in tension, is multiplied by Equation (17) and considering Equations (16) and (10a) results in

$$F_i\left(-\beta-\frac{r^2}{y_t}\right)<\sigma_{ti}\,Z_t+M_{DL} \tag{19}$$

If $\left(-\beta-\frac{r^2}{y_t}\right)>0$ the inequality in Equation (19) becomes

$$F_{i\max}=\frac{\sigma_{ti}\,Z_t+M_{DL}}{\left(-\beta-\frac{r^2}{y_t}\right)} \tag{20a}$$

If $\left(-\beta-\frac{r^2}{y_t}\right)<0$, the inequality in Equation (19) becomes

$$F_{i\min}=\frac{\sigma_{ti}\,Z_t+M_{DL}}{\left(-\beta-\frac{r^2}{y_t}\right)} \tag{20b}$$

Similarly from Equation (12b), i.e., when the stress at top fiber is in compression, we can obtain as follows:

If $\left(-\beta-\frac{r^2}{y_t}\right)>0$:

$$F_{i\min}=\frac{\sigma_{ci}\,Z_t+M_{DL}}{\left(-\beta-\frac{r^2}{y_t}\right)} \tag{21a}$$

If $\left(-\beta-\frac{r^2}{y_t}\right)<0$:

$$F_{i\max}=\frac{\sigma_{ci}\,Z_t+M_{DL}}{\left(-\beta-\frac{r^2}{y_t}\right)} \tag{21b}$$

(B). BASED ON ALLOWABLE STRESS AT THE BOTTOM FIBER

On the other hand, from the condition of stress at the bottom fiber at transfer in Equation

(13a) is multiplied by Equation (18) and by using Equations (16) and (10a) we can obtain as follows:

If $\left(\beta - \frac{r^2}{y_b}\right) > 0$:

$$F_{i\max} = \frac{\sigma_{ti} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)} \qquad (22a)$$

If $\left(\beta - \frac{r^2}{y_b}\right) < 0$:

$$F_{i\min} = \frac{\sigma_{ti} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)} \qquad (22b)$$

Similarly, from Equation (13b) we can obtain as follows:

If $\left(\beta - \frac{r^2}{y_b}\right) > 0$:

$$F_{i\min} = \frac{\sigma_{ci} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)} \qquad (23a)$$

If $\left(\beta - \frac{r^2}{y_b}\right) < 0$:

$$F_{i\max} = \frac{\sigma_{ci} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)} \qquad (23b)$$

5. PRESTRESSING FORCE BASED ON FINAL CONDITIONS (AFTER LOSS OF PRESTRESS)

(A). BASED ON ALLOWABLE STRESS AT THE TOP FIBER

If Equation (14a), i.e., when the stress at top fiber is in tension, is multiplied by Equation (17) and considering Equations (16), (10b) and (11) results in

If $\left(-\beta-\frac{r^2}{y_t}\right)>0$:

$$F_{i\max}=\frac{\sigma_t Z_t+M_{TL}}{\alpha\left(-\beta-\frac{r^2}{y_t}\right)} \tag{24a}$$

If $\left(-\beta-\frac{r^2}{y_t}\right)<0$:

$$F_{i\min}=\frac{\sigma_t Z_t+M_{TL}}{\alpha\left(-\beta-\frac{r^2}{y_t}\right)} \tag{24b}$$

Similarly from Equation (14b), when the stress condition is in compression, we can obtain:

If $\left(-\beta-\frac{r^2}{y_t}\right)>0$

$$F_{i\min}=\frac{\sigma_c Z_t+M_{TL}}{\alpha\left(-\beta-\frac{r^2}{y_t}\right)} \tag{25a}$$

If $\left(-\beta-\frac{r^2}{y_t}\right)<0$:

$$F_{i\max}=\frac{\sigma_c Z_t+M_{TL}}{\alpha\left(-\beta-\frac{r^2}{y_t}\right)} \tag{25b}$$

(B). BASED ON ALLOWABLE STRESS AT THE BOTTOM FIBER

On the other hand from the stress at the bottom fiber at the final stage in Equation (15a), i.e., when the stress is in tension, is multiplied by Equation (18) and considering Equations (16), (10b) and (11) results in as follows:

If $\left(\beta-\frac{r^2}{y_b}\right)>0$:

$$F_{i\max} = \frac{\sigma_t Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)} \tag{26a}$$

If $\left(\beta - \frac{r^2}{y_b}\right) < 0$:

$$F_{i\min} = \frac{\sigma_t Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)} \tag{26b}$$

Similarly from Equation (15b) we can obtain as follows:

If $\left(\beta - \frac{r^2}{y_b}\right) > 0$:

$$F_{i\min} = \frac{\sigma_c Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)} \tag{27a}$$

If $\left(\beta - \frac{r^2}{y_b}\right) < 0$:

$$F_{i\max} = \frac{\sigma_c Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)} \tag{27b}$$

6. APPLICATION

Equations (20)-(27) may be used to define the range of prestressing force F_i to satisfy the stress conditions at the initial and final stage (after loss of prestress). It is to be noted that the resulting prestressing force will satisfy the stress conditions in Equations (12)-(15) and alleviate the use of trial and error or designing concordant cable due to the presence of secondary moment in statically indeterminate structures. When optimum design is desired theoretically $F_{imax} = F_{imin}$ or the difference of prestressing force F_{imax} and F_{imin} obtained from Equations (20)-(27) should be minimum. However, if F_{imax} is smaller than F_{imin} means that the section is too small.

It is to be noted also that the equations derived in this paper can be used for both statically determinate and indeterminate structures where the coefficient β in Equations (10a) and (10b) equals the cable eccentricity e since the secondary moment is equal to zero. The coefficient β here can be viewed as ‘statically indeterminate eccentricity’ because in statically determinate structures the moment due to prestressing equals to the force times eccentricity. In addition β can be viewed as moment coefficient (influence) because when F or F_i equals to unity the moment in Equations (10a) and (10b) equals to β.

In multi-storey buildings when the magnitude of the prestressing force in the beam may be different from one floor to another (or event might be different from beam to beam for a particular floor), the resulting equations may still be used to obtain the prestressing force provided that the ratio of prestressing force are known for every beam. In this case the ratio of prestressing force can be estimated based on the external load to be carried by each beam in the structures.

Because it is uneconomical to design the prestressing force to satisfy all loading conditions, usually the prestressing force is obtained based on gravity load only. For structures built in earthquake zones, where there are load reversal applied to the members, ordinary (non-prestressed) reinforcements are needed. In this case

$$\phi M_n \geq M_u \tag{28}$$

where ϕ = strength reduction factor, M_n = nominal (resistance) moment including prestressed and non-prestressed reinforcement, and M_u = ultimate moment due to external load. For statically indeterminate structures, moment due to external load should include the secondary moments from prestressing force with load factor is set as unity

$$M_u = \gamma_D M_{DL} + \gamma_L M_{LL} + \gamma_E M_E + \gamma_s M_s \tag{29}$$

where $\gamma_D, \gamma_L, \gamma_E$ are load factors for dead, live and earthquake loads stipulated by building codes, respectively, while γ_s is load factor for the effect of prestressing force. The value of γ_s is taken as to 1. M_{DL}, M_{LL} and M_s are as before and M_E is moment due to earthquake.

Consider now a nine-story building as shown in Figure 7. The dimension and property of beams are shown in Table 1. The structure is subjected to dead, live and earthquake loads as shown in Table 2. Concrete strength $f_c^{'}$ = 30 MPa, at transfer concrete strength, $f_{ci}^{'}$ = 25 MPa. The allowable stresses are as follows:

at transfer: $\sigma_{ci} = -0.6 f_{ci}^{'}$ and $\sigma_{ti} = 0.25\sqrt{f_{ci}^{'}}$;

at final condition: $\sigma_c = -0.45 f_c^{'}$ and $\sigma_t = 0.5\sqrt{f_c^{'}}$.

The tendon's trajectory is taken as double curvatures and at each end has horizontal alignment as shown in Figure 7.

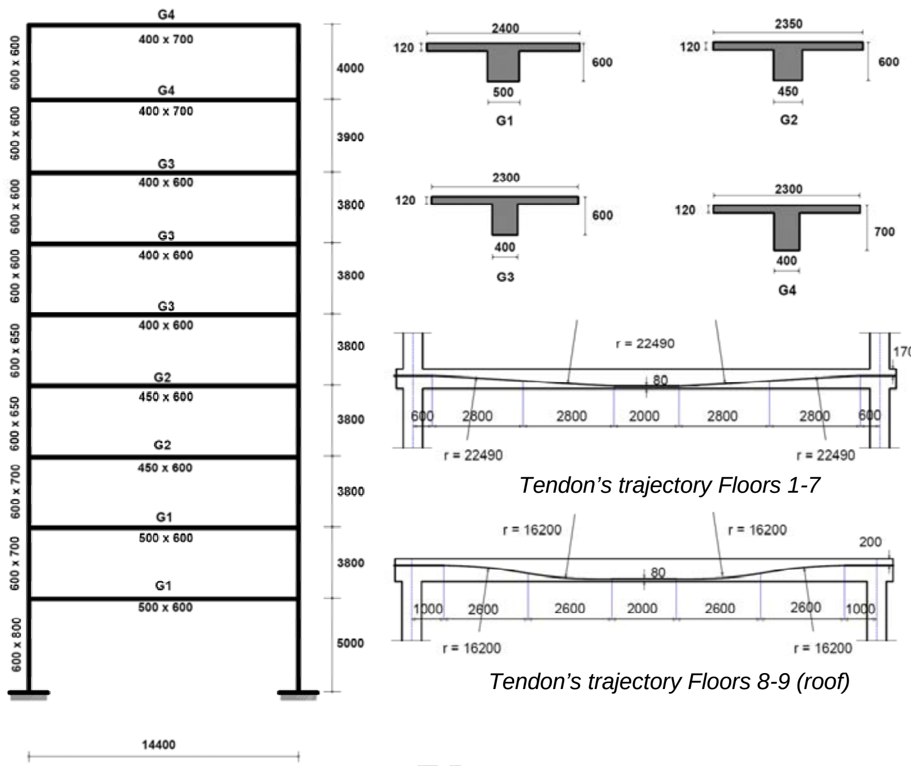

Figure 7. Nine story building

Table 1: Beam properties

Floor	B (m)	H (m)	Flange width b_f (m)	Flange thickness t(m)	y_a (m)	y_b (m)	A_c (m^2)	I_c (m^4)	Cable eccentricity at beam's end e (m)
9	0.45	0.70	2.35	0.12	0.23011	0.47177	0.543	0.024260	0.028230
8	0.45	0.70	2.35	0.12	0.23011	0.47177	0.543	0.024260	0.028230
7	0.40	0.60	2.30	0.12	0.18308	0.41692	0.468	0.014208	0.013077
6	0.40	0.60	2.30	0.12	0.18308	0.41692	0.468	0.014208	0.013077
5	0.40	0.60	2.30	0.12	0.18308	0.41692	0.468	0.014208	0.013077
4	0.45	0.60	2.35	0.12	0.19012	0.40988	0.498	0.015494	0.020120
3	0.45	0.60	2.35	0.12	0.19012	0.40988	0.498	0.015494	0.020120
2	0.50	0.60	2.40	0.12	0.19636	0.40364	0.528	0.016735	0.026364
1	0.50	0.60	2.40	0.12	0.19636	0.40364	0.528	0.016735	0.026364

Table 2: Dead, live and earthquake loads

Lantai	**q_{DL} (kN/m)**	**q_{LL} (kN/m)**	**E (kN)**
9	25.43	3.60	69.52
8	19.10	5.40	63.56
7	18.12	5.40	52.46
6	18.12	5.40	44.89
5	18.12	5.40	39.68
4	18.70	5.40	35.21
3	18.70	5.40	29.90
2	19.30	5.40	23.07
1	19.30	5.40	12.40

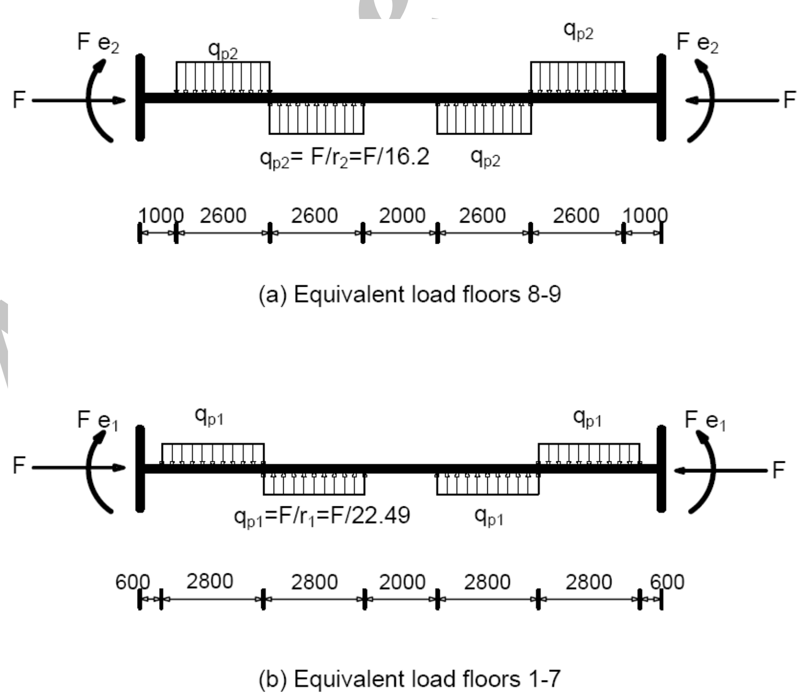

Figure 8. Equivalent load due to prestressing force

Due to the tendon's trajectory, equivalent load applied to members due to prestressing force is depicted in Figure 8 in which for Floors 1-7:

$$q_{p1} = F\,\theta_1/l_1 = F/r_1 = 0.0445\,F$$

where $\theta_1 \approx tg\,\theta_1 \approx \sin\theta_1 = l_1/r_1$, r_1 = 22490 mm = 22.49 m and l_1 = 2800 mm.

Similarly equivalent load for Floors 8-9:

$$q_{p2} = F/r_2 = 0.0617\,F$$

where r_1 = 16200 mm = 16.2 m.

Structural analysis is done. In order to obtain coefficient β prestressing force is taken as a unit force. Results of structural analysis are presented in Tables 3 and 4.

Table 3: Midspan moment for beams and coefficient β

Floor No	**M_{DL} (kNm)**	**M_{LL} (kNm)**	**M_{TL} (kNm)**	**β (m)**
9	315.269	42.577	357.846	-0.246
8	180.907	54.280	235.187	-0.221
7	175.599	52.017	227.616	-0.175
6	174.481	52.044	226.525	-0.175
5	172.159	51.348	223.507	-0.173
4	177.630	51.237	228.866	-0.173
3	175.782	50.812	226.595	-0.172
2	180.778	50.533	231.311	-0.172
1	183.314	51.297	234.611	-0.168

To obtain the prestressing force, Equations (20) – (27) are used, where loss of prestress is taken 20% such that α = 0.80. In this case it is assumed that prestressing force at each floor will be the same. However, if needed the prestressing force at each floor can be different by assigning a ratio from floor to floor. From the results of Equations (20) – (27) and as shown in Table 5, F_{imin} = 796.21 kN and F_{imax} = 2766.8 kN. The value of prestresing force at transfer can be taken as the average value, i.e., F_i = 1780 kN. Effective prestress after transfer F = 0.80 x 1780 = 1424 kN.

Table 4: Left-end moment for beams and coefficient β

Floor No	M_{DL} (kNm)	M_{LL} (kNm)	M_{TL} (kNm)	β (m)	M_E (kNm)
9	–343.093	–50.624	–393.717	0.171	104.225
8	–313.576	–85.522	–399.098	0.196	225.154
7	–293.513	–87.784	–381.298	0.173	283.522
6	–294.631	–87.758	–382.389	0.174	383.703
5	–296.952	–88.454	–385.406	0.175	460.601
4	–306.498	–88.565	–395.063	0.176	542.627
3	–308.345	–88.989	–397.335	0.177	576.328
2	–318.884	–89.268	–408.152	0.177	603.456
1	–316.347	–88.505	–404.852	0.181	500.025

Table 5: Prestressing force determination

Floor No	Left end		Midspan	
	F_{imin} (kN)	F_{imaks} (kN)	F_{imin} (kN)	F_{imaks} (kN)
9	578.68	5270.3	796.21	3189.4
8	465.94	3730.1	373.60	3016.3
7	622.62	3355.0	674.25	2771.3
6	624.80	3332.9	671.85	2766.8
5	634.03	3323.0	661.97	2779.9
4	632.56	3534.2	629.47	2991.7
3	639.04	3517.5	620.56	2996.3
2	645.48	3764.4	587.61	3204.0
1	625.89	3592.7	613.88	3266.3
Result	F_{imin} = 796.21 kN F_{imax} = 2766.80 kN			

In order to consider earthquake loading as an example consider a beam in the third floor where the section is shown in Figure 9:

$$M_u = \gamma_D M_{DL} + \gamma_L M_{LL} - \gamma_E M_E = -1053.332 \text{ kNm, for earthquake from right.}$$

Secondary moment at left end: $M_s = M_F$ – primary moment = 252.048 – 28.651 = 223.397 kNm, where M_F is taken from Equation (10b) and primary moment is prestressing force multiplied by eccentricity. For statically indeterminate structures:

$$M_u = \gamma_D M_{DL} + \gamma_L M_{LL} - \gamma_E M_E + \gamma_s M_s = -1052.332 + 223.397 = -828.835 \text{kNm, where}$$

$\gamma_s = 1$.

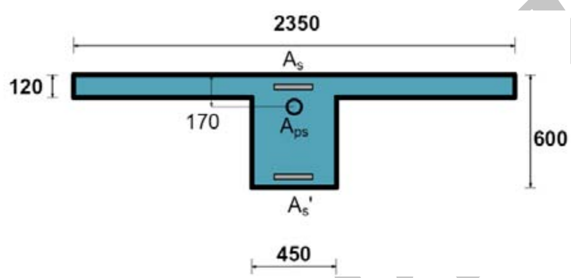

Figure 9. Beam section at left end with non-prestressed reinforcement

The non-prestressed reinforcement can be obtained by following the design requirements stipulated in building codes such as in ACI 318-08 [6]. For simplicity, nominal moment due to prestressing can only be determined by assuming tension and compression reinforcement are the same such that the effective stress in prestressing can be determined from the approximation. The difference between $\frac{M_u}{\phi}$ and nominal moment due to prestressing has to be resisted by non-prestressed reinforcements A_s and $A_s^{'}$ for tension and compression reinforcement, respectively. Prestressing strands can be taken as 10 x Ø15,2 mm = 10 x 140 = 1400 mm^2, f_{pu} = 270 ksi = 1860 MPa, f_{py} = 1670 MPa and non-prestressed reinforcement f_y = 400MPa. From this result $A_s = A_s^{'}$ = 4D25.

7. CONCLUSIONS

The determination of prestressing force in statically indeterminate structures is discussed in this paper. Equations are derived so that the magnitude of the prestressing force can be obtained directly from those equations. This is achieved by assuming that the value of moment due to prestressing force is represented by a moment coefficient β by assigning prestresing force as a unit force. By using moment coefficient β it is not necessary to consider the secondary moment since the effect of prestressing in statically indeterminate structures has been included in the calculation. Equations to obtain prestressing force are derived from the stress requirements. It is to be noted that the derived equation can be used for both statically determinate and indeterminate structures. In statically determinate structures the moment coefficient β turns to be cable eccentricity e since the primary moment equals to the moment due to equivalent load. Therefore, β can be viewed as the 'statically indeterminate eccentricity', i.e., the 'eccentricity' in statically indeterminate structures. By assigning prestressing force as a unit force, structural analysis due to equivalent load can be done. By doing this the prestressing force can be obtained easily by using the derived equation. It is to be noted that by using the method proposed in this paper it is not necessary to make tendon transformation in order to have zero secondary moments.

An example on how to apply the procedure is presented to a nine-story building, where the prestressing force can be easily obtained from the derived equations. For structures with load reversal such as structures in earthquake zones it is uneconomical to design the prestressing force to satisfy all loading conditions. Therefore, to resist earthquake loading non-prestressed reinforcements are needed. The determination of non-prestressed reinforcement can be done following the provisions in building codes.

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